tance changes in excess of that predicted from the inverse-cube-root variation of the depletion-layer capacitance. This is attributed to stored charge carriers under conditions of forward bias. Thus the harmonic generation efficiency of these diodes is above that predicted for cube-root capacitance variation. However, it is interesting to note that agreement between experiment and theory can be obtained for at least one published result using these diodes,[5] using Fig 4 of Hyltin and Kotzebue which is for square-root capacitance variation. The result is that of Lowell and Kiss who reported on a fifth-harmonic and eighth-harmonic generator. Using their data on the diodes used, and assuming that the effective diode capacitance at operating bias is 0.6 the zero bias values, we obtain predicted efficiencies of about 5 db for the fifth harmonic circuit and about 19 db for the eighth harmonic circuit. The values reported by Lowell and Kiss are aout 5.5 db and 19 db, respectively.

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[5] R. Lowell and M. J. Kiss, "Solid-state microwave power sources using harmonic generation," Proc. IRE, vol. 48, pp. 1334-1335; July, 1960.

Double-Layer Matching Structures*

In the millimeter wavelength region, as in other regions, the design of matching layers for dielectric surfaces is limited by the lack of suitable dielectric materials. Artificial dielectric layers, formed by periodic perturbations of the boundary surface, are not practical because of the small physical dimensions required. The following approach uses two layers of materials whose relative dielectric constants are given. The thicknesses of the layers are chosen to eliminate reflections at the desired center frequency. A broad-band match is obtained because the layers can be made less than an eighth wavelength in thickness.

In the case of normal incidence upon lossless dielectric layers, transmission line theory may be used to determine the matching conditions. Refering to Fig. 1, and assuming that the first section is matched,

$$Z_4 = Z_3 \frac{Z_2(Z_1 + jZ_2 \tan\theta_2) + jZ_3 \tan\theta_3(Z_2 + jZ_1 \tan\theta_2)}{Z_3(Z_2 + jZ_1 \tan\theta_2) + jZ_2 \tan\theta_3(Z_1 + jZ_2 \tan\theta_2)} \qquad (1)$$

at the center frequency. The characteristic impedance and electrical length of the jth section are Z_j and θ_j, respectively.

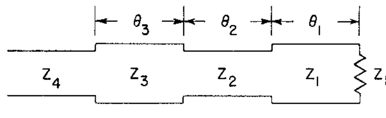

Fig. 1—Cascaded transmission line system.

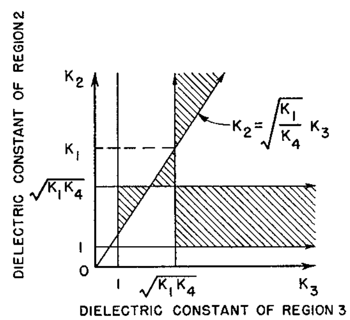

Fig. 2—Allowed values of relative dielectric constants of regions 2 and 3.

Equating real and imaginary parts of (1) gives two equations which may be solved for θ_2 and θ_3

$$\theta_2 = \tan^{-1}\left[\frac{n_2^2(n_1n_4 - n_3^2)(n_4 - n_1)}{(n_2^2 - n_1n_4)(n_2^2n_4 - n_1n_3^2)}\right]^{1/2} \qquad (2)$$

$$\theta_3 = \tan^{-1}\left[\frac{n_3^2(n_2^2 - n_1n_4)(n_4 - n_1)}{(n_1n_4 - n_3^2)(n_2^2n_4 - n_1n_3^2)}\right]^{1/2}. \qquad (3)$$

The refractive index of the jth layer is $n_j = \sqrt{K_j}$, where K_j is the relative dielectric constant of the layer.

In a typical problem, K_1 and K_4 are specified. It may be shown that for $K_1 > K_4$, the values of K_2 and K_3 that yield real values of θ_2 and θ_3 lie in the shaded regions shown in Fig. 2. A practical design can usually be found using only available low-loss dielectric materials.

As an example, consider the problem of matching a polystyrene-air interface. Letting $K_1 = 2.56$ for polystyrene and $K_4 = 1$ for air, it is seen that one solution is

$$K_2 = 1.00$$

$$K_3 = 2.56$$

$$\theta_2 = \theta_3 = 29.1°.$$

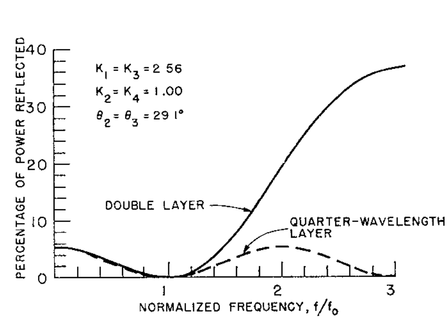

Fig. 3—Reflected power as function of normalized frequency.

Fig. 3 shows ratio of reflected power to incident power for this solution and for a quarter-wavelength layer designed for the same center frequency. The bandwidths of the two structures are comparable.

Since thin films of most plastics are commercially available, the double layer matching structure is feasible at millimeter wavelengths. One such structure using polystyrene film and polystyrene foam sheet on polystyrene has proved successful at 70 Gc.

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Microwave Noise-Figure Measurement for Small Noise Output*

The purpose of this communication is to propose a new method of noise-figure measurement for a microwave amplifier of small noise output. The new method is helpful when the accuracy of a conventional method is not satisfactory.

In conventional measurements, when the noise output of the microwave amplifier is too small for direct noise power measurement, an auxiliary receiver is used after the amplifier under test. The noise figure of the amplifier, F_1, is given by[1]

$$F_1 = F_{12} - \frac{F_2 - 1}{G_1}. \qquad (1)$$

In this equation,

F_{12} = noise figure of over-all system,
F_2 = noise figure of the auxiliary receiver,
G_1 = gain of the amplifier under test.

It is possible to express F_1, F_2, G_1, and F_{12} in the following manner:

$$\left.\begin{array}{l} F_1 \equiv f_1 \times 10^{n_1},\ F_2 \equiv f_2 \times 10^{n_2}, \\ G_1 \equiv g \times 10^{m} \end{array}\right\} \qquad (2)$$

and

$$F_{12} \equiv f_{12} \times 10^{n_{12}}$$

where $0 < (f_1, f_2, f_{12} \text{ or } g) > 10$ and n_1, n_2, m and n_{12} are positive integers. Then, when $F_2 \gg 1$, (1) can be rewritten as

$$F_1 \doteq f_{12} \times 10^{n_{12}} - \frac{f_2}{g} \times 10^{n_2 - m}$$

$$= \left(f_{12} - \frac{f_2}{g} \times 10^{n_2 - m - n_{12}}\right) \times 10^{n_{12}} \qquad (3)$$

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[1] H. T. Friis, "Noise figures of radio receivers," Proc. IRE, vol 32, pp. 419-422; July, 1944.